A Unified Approach for Mixed EM and Circuit Simulation Using Model-Reduction Techniques

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Abstract - *With the continually increasing operating speeds, the boundary between EM problems and general circuit designs is narrowing rapidly. There is a great demand for efficiently combining EM solutions with general purpose circuit simulators such as SPICE. In this paper we present a novel and a unified approach based on model-reduction techniques to combine EM formulations such as FEM, directly into general purpose circuit simulators. The method is two to three order faster than the previously published methods and is suitable for simulating large number of electromagnetic devices in a general circuit environment consisting of lumped/distributed circuit elements and nonlinear terminations.*

I - Introduction

The trend in the microwave and VLSI industries is to design faster circuits and to integrate many functions in a single design. Consequently, designs are made with higher operating speeds, sharper rise times, shrinking device sizes and low power consumption. These aspects are making the signal integrity analysis a challenging task and are highlighting the interconnect effects, such as ringing, signal delay, distortion, reflections and crosstalk [1]. At very high frequencies lumped and distributed interconnect models based on quasi-TEM approximations will become inaccurate and sophisticated full-wave EM models which take into account all possible field components and all boundary conditions will be required. This needs the simulation of large number of interconnect EM models along with other lumped linear and nonlinear components. Also on a parallel front, there is a need for simulating lumped components such as resistors, capacitors and inductors etc. along with EM devices in a nonlinear environment. Both the above mentioned issues which are currently challenging the high-frequency circuit designs demand an efficient technique for unified or global simulation of EM and lumped/distributed components.

There have been few attempts in the literature to address the above issue. These approaches can be classified into two categories. In the first category time-domain global simulation is performed based on the extension of FDTD techniques to include lumped components [2], [3]. However, such an approach suffers in a general circuit environment containing stiff systems due to numerical instability [4]. Approaches in the second category are based on the steady state frequency-domain techniques such as harmonic balance method [5]. However, extension of this idea to obtain general transient solutions by directly coupling FEM equations with nonlinear circuits leads to a large set of differential equations, solution of which will be prohibitively CPU expensive. In addition, in many practical applications because of the intricate frequency dependence in FEM equations (e.g. hybrid FEM/boundary integral systems) [6], it may not be possible to combine them directly with nonlinear differential equations. Hence there is a need for an efficient and accurate technique for global simulation of entire circuit comprising of lumped, EM and nonlinear components.

In this paper, we describe a novel method to address the global simulation of EM, lumped linear and nonlinear components efficiently. The main contributions in the new technique are summarized below:

(1) The modified nodal analysis (MNA) [7] is extended to include FEM formulations and a new stencil for FEM analysed EM devices is derived.

(2) An algorithm for model-reduction of linear portion of the global ciruit is presented to reduce the size of the problem under consideration.

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(3) A new minimum realization algorithm is developed to obtain a macromodel in terms of minimal-order state-space representation from the reduced-order description. Proposed algorithm guarantees the number of extra states required during macromodeling to be minimum while retaining the accuracy.

II - Formulation of Circuit Equations

Consider a general nonlinear network ϕ which contains linear lumped components, EM components and arbitrary linear subnetworks. The linear lumped components may be described by equations in either time-domain or frequency-domain, whereas the nonlinear components may only be described by time-domain equations. The arbitrary linear subnetworks may contain EM devices that are best described in the frequency-domain. Let the linear subnetworks be grouped into a single subnetwork π. Without loss of generality, the modified nodal admittance (MNA) matrix [7] for the network ϕ can be formulated as

$$\boldsymbol{C}_{\phi}\frac{d}{dt}\boldsymbol{v}_{\phi}(t) + \boldsymbol{G}_{\phi}\boldsymbol{v}_{\phi}(t) + \boldsymbol{D}_{\pi}\boldsymbol{i}_{\pi}(t) + \boldsymbol{F}(\boldsymbol{v}_{\phi}(t)) - \boldsymbol{b}_{\phi}(t) = 0, \qquad t \in [0,T] \quad \textbf{(1)}$$

where

- $\boldsymbol{v}_{\phi}(t) \in \Re^{N_{\phi}}$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,
- $\boldsymbol{C}_{\phi} \in \Re^{N_{\phi} \times N_{\phi}}$ and $\boldsymbol{G} \in \Re^{N_{\phi} \times N_{\phi}}$ are constant matrices describing the lumped memory and memoryless elements of network ϕ, respectively,
- $\boldsymbol{b}_{\phi} \in \Re^{N_{\phi}}$ is a constant vector with entries determined by the independent voltage and current sources,
- $\boldsymbol{F}(\boldsymbol{v}_{\phi})$ is a function describing the nonlinear elements of the circuit,
- $\boldsymbol{D}_{\pi} = [d_{i,j} \in \{0, 1\}], i \in \{1, \ldots, N_{\phi}\}, j \in \{1, \ldots, n_{\pi}\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $\boldsymbol{i}_{\pi}(t) \in \Re^{n_{\pi}}$, the vector of currents entering the linear subnetwork π, into the node space $\Re^{N_{\phi}}$ of the network ϕ, N_{ϕ} is the total number of variables in the MNA.

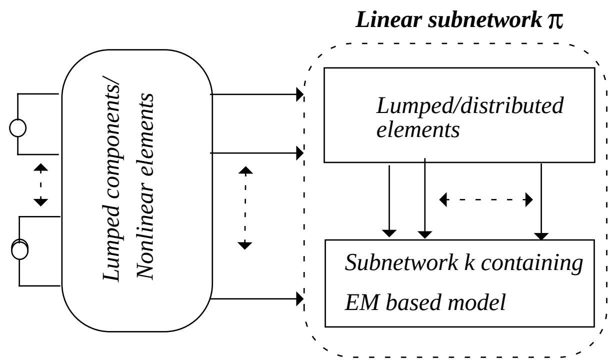

Fig. 1. Nonlinear Network ϕ containing linear subnetworks with lumped and EM components

The linear multi-terminal subnetwork π can be characterized in the frequency-domain by its terminal behavior. Without loss of generality, the terminal relations for subnetwork π can be represented by frequency-domain equations in the form (described in section C)

$$\boldsymbol{I}_{\pi}(s) = \boldsymbol{Y}_{\pi}(s)\boldsymbol{V}_{\pi}(s) \quad \textbf{(2)}$$

where s is the complex frequency, $\boldsymbol{Y}_{\pi}(s)$ is the complex frequency-domain admittance representation of the subnetwork π, $\boldsymbol{V}_{\pi}(s)$ is the vector of terminal voltage nodes that connects subnetwork π to network ϕ and $\boldsymbol{I}_{\pi}(s) = L(\boldsymbol{i}_{\pi}(t))$; L denotes the Laplace transform.

The difficulty in solving (1) and (2) simultaneously is due to the fact that they implicitly contain a mixture of frequency- and time-domain representations. This can be efficiently addressed using the following three basic steps:

1) Using moment-matching techniques, $\boldsymbol{Y}_{\pi}(s)$ in (2) is approximated by a q- pole lower-order model.

2) Using q- pole lower-order model, a *minimal-order* state-space representation in the time-domain is derived.

3) The derived differential equations are solved simultaneously with (1) using standard numerical techniques or any general-purpose circuit simulators.

III - MNA formulation of linear subnetworks containing EM models

In order to perform model-reduction on a linear subnetwork comprising of lumped components and

EM models as needed by step #1, MNA representation of the linear subnetwork π is required. This is accomplished by extending the modified nodal analysis to include EM models and also by deriving a generalized stencil for FEM based formulations. Because of space limitations only the final form of the MNA is given below and is presented for the case of network ϕ containing a single linear network with a single EM model.

$$\begin{bmatrix} s\boldsymbol{W}+\boldsymbol{G} & \boldsymbol{P} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F} & s^2\boldsymbol{A}+s\boldsymbol{B}+\boldsymbol{C} \\ \boldsymbol{K} & \boldsymbol{0} & \boldsymbol{Q} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}(s) \\ \boldsymbol{I^e}(s) \\ \boldsymbol{X}(s) \end{bmatrix} = \begin{bmatrix} \boldsymbol{J} \\ \boldsymbol{R}(s) \\ \boldsymbol{0} \end{bmatrix} \quad \textbf{(3)}$$

where

- $\boldsymbol{C}, \boldsymbol{G} \in \Re^{N_\pi \times N_\pi}$ are constant matrices determined by lumped linear components of subnetwork π; $\boldsymbol{J} \in \Re^{N_\pi}$ is a constant vector with entries determined by independent voltage and current sources of subnetwork π,
- $\boldsymbol{V}(s) \in \Re^{N_\pi}$ is the vector of Laplace-domain node voltage waveforms appended by independent voltage source current, linear inductor current waveforms of linear subnetwork π,
- $\boldsymbol{I^e} \in \Re^{N^e}$ is the vector containing Laplace-domain terminal currents entering the EM based subnetwork; N^e is the number of terminals in EM based subnetwork,
- $\boldsymbol{X}(s)$ is a vector of unknown field components from FEM formulation of EM based subnetwork,
- ***A, B, C*** are matrices derived in terms of nodal/edge based FEM elemental matrices of EM based subnetwork,
- $\boldsymbol{P}$ and $\boldsymbol{K}$ are binary incidence matrices that map $\boldsymbol{I^e} \in \Re^{N^e}$ into the node space $\Re^{N_\pi}$ of subnetwork π and terminal voltages of EM subnetwork into node space $\Re^{N_\pi}$ of subnetwork π, respectively,
- $\boldsymbol{Q}$ is a functional matrix which relates the unknown filed components $\boldsymbol{X}(s)$ to terminal voltages of EM based subnetwork,
- $\boldsymbol{F}$, $\boldsymbol{R}(s)$ are matrices obtained by incorporating boundary conditions associated with the EM based subnetwork.

IV - Time-domain Macromodel of $\boldsymbol{Y}_\pi(s)$ Through Model-Reduction

Generally, solution of equations represented by (3) is highly CPU intensive. In addition, simultaneous solution of (1) and (2) requires time-domain representation for $\boldsymbol{Y}_\pi(s)$. Application of model-reduction techniques such as complex frequency hopping (CFH) [8] solves both these problems. Using (3) a q- pole lower order model for $\boldsymbol{Y}_\pi(s)$ can be obtained as:

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n_\pi} \\ \dots & \dots & \dots & \dots \\ Y_{n_\pi 1} & Y_{n_\pi 2} & \dots & Y_{n_\pi n_\pi} \end{bmatrix} \begin{bmatrix} V_1 \\ \dots \\ V_{n_\pi} \end{bmatrix} = \begin{bmatrix} I_1 \\ \dots \\ I_{n_\pi} \end{bmatrix} \quad \textbf{(4)}$$

$$Y_{jk}(s) = d^{j,k} + \sum_{i=1}^{q^{j,k}} \frac{r_i^{j,k}}{s - p_i^{j,k}}; \qquad 1 \le (j,k) \le n_\pi$$

Moments of FEM based components required in model-reduction can be calculated as described by [9].

Using q- pole lower-order description (4) a *minimal-order* state-space representation in the time-domain is derived as

$$\begin{aligned} [\dot{\boldsymbol{Z}}] &= [\boldsymbol{A}][\boldsymbol{Z}] + [\boldsymbol{B}][\boldsymbol{i}_\pi] \\ [\boldsymbol{v}_\pi] &= [\boldsymbol{C}][\boldsymbol{Z}] + [\boldsymbol{D}][\boldsymbol{i}_\pi] \end{aligned} \quad \textbf{(5)}$$

where $\boldsymbol{i}_\pi$ and $\boldsymbol{v}_\pi$ are the vector of terminal currents and voltages of linear subnetwork π. Gilbert's diagonal construction technique [10] is used to derive a macromodel which guarantees a "minimal-order state-space representation" for the multiport transfer function represented by (4). The macromodel thus derived is both controllable and observable.

The differential equations represented by the macromodel (5) can now be combined with (1) making use of the relation $\boldsymbol{v}_\pi = (\boldsymbol{D}_\pi)^t \boldsymbol{v}_\phi$. Using standard nonlinear solvers, resulting unified set of differential equations can be solved to yield accurate global transient solutions for the entire nonlinear circuit [11].

V - Computational Results

An example is given below (Fig. 2) to demonstrate the merits of the proposed technique. The linear network contained a single high-speed interconnect. To verify the accuracy of the proposed global simulation technique, cross-sectional dimensions of the interconnect are chosen such that they are relatively small compared to the signal wavelength. Under such an assumption a quasi-TEM approximation would be adequate for comparing the results. An EM analysis is carried on the interconnect using FEM formulation and from the resulting stencil a global circuit matrix (MNA) (3) is obtained. CFH is applied on this MNA to obtain a reduced-order model, and subsequently a time-domain macromodel for the entire linear network is derived. In Fig. 3 time responses obtained using both the proposed technique (with FEM-based interconnect model) as well as SPICE (with quasi-TEM interconnect model) are given. As seen, they match within reasonable accuracy.

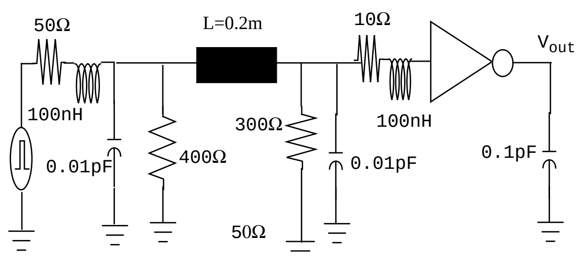

Fig. 2. **Nonlinear circuit containing interconnect with EM model**

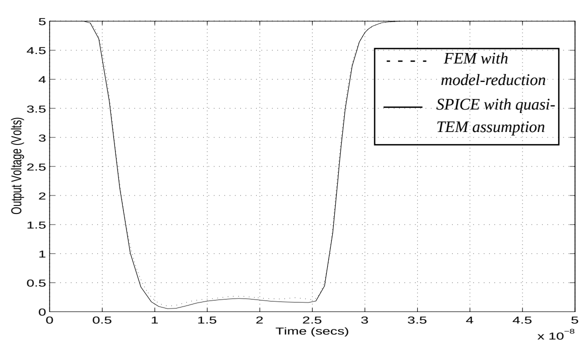

Fig. 3. **Time response at output V_{out}**

VI - Conclusions

In this paper an efficient technique based on model-reduction is presented for global simulation of circuits containing EM devices, lumped linear and nonlinear components. A generalized stencil for EM devices with FEM formulation is derived for inclusion in MNA analysis. Also a new minimum realization algorithm is used for macromodel synthesis. The proposed technique an provides an efficient means for mixed frequency/time simulation.

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